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The Principle of Kinematics and Power

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***Abstract* – These there are dynamic principles both in particles and rigid bodies such as Principle of Work and Energy and Principle of Impulse and Momentum. During the past few year, there are many researches about Jerk in both direct (the third derivative of displacement with respect to time) and indirect (the first derivative of force with respect to time) jerks. This paper describes about how the Principle of Kinematics and Power according to particles and rigid bodies are derived and the meaningful of the principle via examples.**

***Keywords* – Principle, Rigid Body, Kinematics, Power, Indirect Jerk.**

I. Introduction

When solving dynamic problem according to particles, the Newton's second law is applied to determine the change of velocity or displacement of particle via integration of the acceleration using appropriated kinematic equations. Still there are classes of problems in which the change of unbalanced forces with respect to time (indirect jerk) acting on particles are of interest [2], [3]. The integration of the indirect jerk with respect to the displacement of particle is concerned as power that applied on particle during the corresponding displacement. This relationship can be derived as the principle of kinematic and power which are the subject of this paper.

II. Principle Derivation

From Fig.1, the power done by the change of force in time $\frac{d\vec{F}}{dt}$ during the particle motion from point A to point B is defined as

$$dP = \frac{d\vec{F}}{dt}\cdot d\vec{r}\ , \qquad (1)$$

where dP called work which is a scalar quantity.

Now, in order to derive the principle of power and kinematics, the Newton's second law must be taken derivative with respect to time which may be written as

$$\sum \vec{F} = m\vec{a}\ , \qquad (2)$$

where $\vec{a}$ is an acceleration of the particle.

Equation (2) is now taken derivative with respect to time, become

$$\frac{d}{dt}\sum \vec{F} = m\frac{d\vec{a}}{dt}\ . \qquad (3)$$

Defining

$$\vec{J}e = \frac{d}{dt}\sum \vec{F}\ , \qquad (4)$$

called indirect jerk [4], [5], so that (3) becomes

$$\vec{J}e = m\frac{d\vec{a}}{dt}\ . \qquad (5)$$

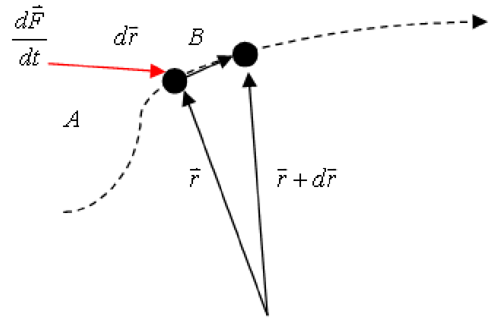

Fig.1. Particle in Motion

During the finite movement of the particle according to a change of force in time, from (1) and (2), the power equal to

$$P = \int_A^B \frac{d}{dt}\sum \vec{F}\cdot d\vec{r}\ . \qquad (6)$$

If (3) is dotted with an infinitesimal displacement, the equation can be written as

$$\frac{d}{dt}\sum \vec{F}\cdot d\vec{r} = m\frac{d\vec{a}}{dt}\cdot d\vec{r}\ . \qquad (7)$$

Rearrange (7) and take integration both sides yield

$$\int_A^B \frac{d}{dt}\sum \vec{F}\cdot d\vec{r} = \int_{a_A}^{a_B} m\vec{v}\cdot d\vec{a}\ . \qquad (8)$$

Finally, (8) is called Principle of Power and Kinematic since the left hand side represents a power and the right hand side contains kinematic quantities. Moreover, this principle can be written in a compact form as

$$\int_A^B \vec{J}e\cdot d\vec{r} = mv\left(a_B - a_A\right). \qquad (9)$$

Consider the plane Kinematics of rigid bodies when a couple M acting on the body, the power is given by

$$P = M\frac{d\theta}{dt} = M\omega\ , \qquad (10)$$

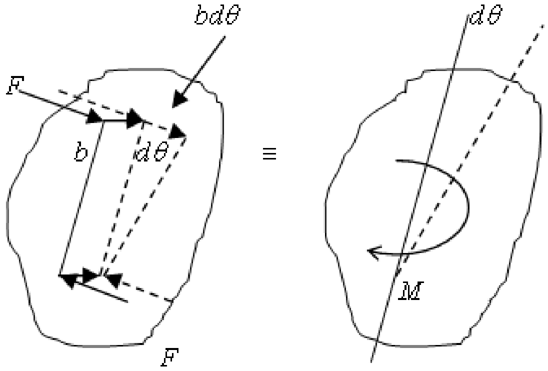

Fig.2. Rigid Body in Motion

where $d\theta$ and ω are the differential angular displacement and the angular velocity, respectively.

If force, F and moment, M are applied to the rigid body, the total power becomes

$$P = \vec{F} \cdot \vec{v} + M\omega \, . \qquad (11)$$

Similarity to the particle derivation of the principle of kinematics and power, the power in Eq. (11) can be rewritten as

$$P = \int_A^B \frac{d}{dt} \sum \vec{F} \cdot d\vec{r} + \int_{\theta_1}^{\theta_2} \frac{d}{dt} \sum \vec{M} \cdot d\vec{\theta} \, . \qquad (12)$$

Fig. 3 shows the general plane motion of a rigid body where G is a center of mass, ω and α are the angular velocity and acceleration respectively, $\vec{v}$ is a linear velocity of a rigid body m_i and v_i are the mass and velocity of a point i on the rigid body.

Follow the derivation from (1) to (7), (8) that relevant for the rigid body in motion can be written as

$$\int_A^B \frac{d}{dt} \sum \vec{F} \cdot d\vec{r} + \int_{\theta_1}^{\theta_2} \frac{d}{dt} \sum \vec{M} \cdot d\vec{\theta} = \int_{a_A}^{a_B} m\vec{v} \cdot d\vec{a} + \int_{\alpha_1}^{\alpha_2} I\omega \, d\alpha \, . \qquad (13)$$

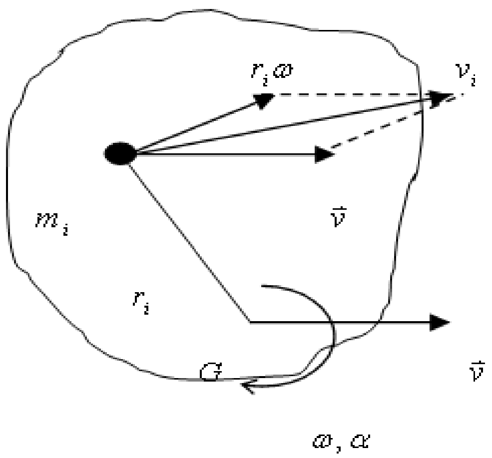

Fig.3. General Plane Motion

Moreover, redefine

$$\vec{J}e = m\frac{d\vec{a}}{dt} + I\frac{d\alpha}{dt} \, , \qquad (14)$$

the Principle of Kinematics and Power can be rewritten as

$$\int_A^B \vec{J}e \cdot d\vec{r} = mv(a_B - a_A) + I\omega(\alpha_B - \alpha_A) \, . \qquad (15)$$

III. Example

The wheel in Fig. 4 rolls up on its inclined hubs without slipping. It is pulled by the force of 200-N which applied to the string wrapped around the outer rim of the wheel. If this wheel starts from rest and its center O has moved up the incline three meters long, compute its angular velocity, ω. The mass of this wheel is 40 Kg and has a radius of gyration of 200 mm. Determine the power input from the force of 200-N and the indirect jerk of the wheels.

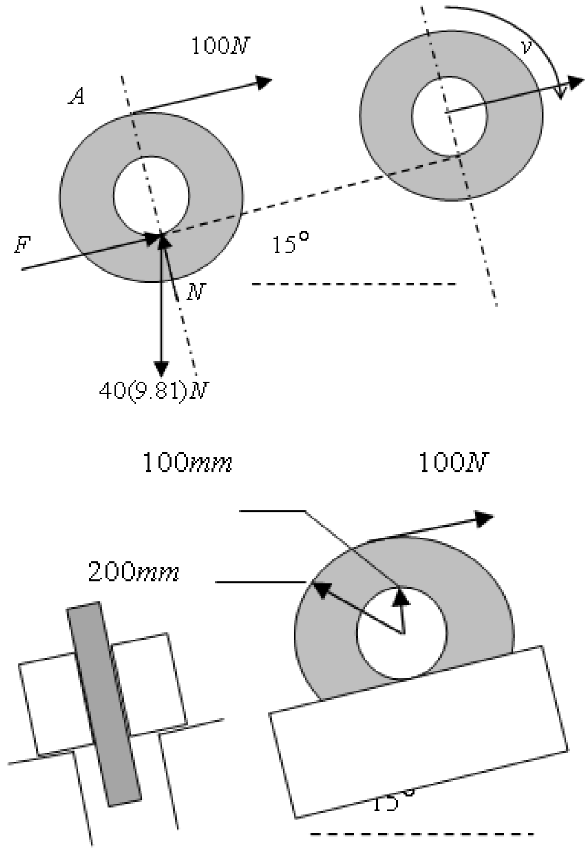

Fig.4. Incline Wheel Free-body Diagram

Of the four forces shown in Fig. 4 the only force 200-N and the weight of the wheel do the work. The friction does not produce the work since there is no slip occurred. From the concept of the instantaneous center of zero velocity, point A has a velocity,

$$v_A = [(200+100)/100]v \, .$$

This because of point A on the string will move $[(200+100)/100] = 3$ times as the point O moves. Then the work done on the wheel is

$$Work = 100\frac{200+100}{100}(3) - (40)(9.81)\sin 15^{O}(3) = 595J \qquad (16)$$

Applying the principle of work and energy, yields,

$$T_1 = 0, \quad T_2 = \frac{1}{2}40(0.1\omega)^2 + \frac{1}{2}40(0.2)^2\omega^2 = 1.0\omega^2 . \qquad (17)$$

Since the equality of (16) and (17), the angular velocity, ω can be computed and it equals to $24.4 rad/s$.

The power input to this wheel is computed by using (11), which is

$$P = 200(0.3)(24.4) = 1464W \, . \qquad (18)$$

The angular acceleration is

$$\alpha_2 = \frac{200(0.3)}{40(0.2)^2} = 37.5 rad/s^2 , \qquad (19)$$

and the acceleration of the wheel after it moves three meters long becomes

$$a_2 = (37.5)(0.1) + (24.4)^2(0.1) = 63.3 m/s^2 . \qquad (20)$$

Finally, the indirect jerk can be obtained by solving (15) as

$$\int_A^B \vec{J}e \cdot d\vec{r} = 40(24.4)(0.1)(63.3) + (40)(0.2)^2(24.4)(63.3) \quad (21)$$

which is

$$\int_0^3 \vec{J}e \cdot d\vec{r} = 8629. \quad (22)$$

Then the indirect jerk, $Je = 2876.33 m/s^3$.

IV. Conclusion

There are dynamic principles both in particles and rigid bodies such as Principle of Work and Energy and Principle of Impulse and Momentum. During the past few year, there are many researches about Jerk in both direct (the third derivative of displacement with respect to time) and indirect (the first derivative of force with respect to time) jerks. This paper describes about how the Principle of Kinematics and Power according to particles and rigid bodies are derived and the meaningful of the principle via examples.

Since there have been several research publications such as [6], [7] and [8], on minimizing dynamic jerk, this principle can be applied and used to analyze more in the area of dynamic optimization [1].

Acknowledgment

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